Methods for Measuring New-Physics Parameters in B Decays

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Abstract

Recently, it was argued that new-physics (NP) effects in B decays can be approximately parametrized in terms of a few quantities. As a result, CP violation in the B system allows one not only to detect the presence of new physics (NP), but also to measure its parameters. This will allow a partial identification of the NP, before its production at high-energy colliders. In this paper, we examine three methods for measuring NP parameters. The first uses a technique involving both $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ penguin B decays. Depending on which pair of decays is used, the theoretical error is in the range 5–15%. The second involves a comparison of $B \to \pi K$ and $B \to \pi\pi$ decays. Although the theoretical error is large ($\gtrsim 25\%$), the method can be performed now, with presently-available data. The third is via a time-dependent angular analysis of $B \to V_1 V_2$ decays. In this case, there is no theoretical error, but the technique is experimentally challenging, and the method applies only to those NP models whose weak phase is universal to all NP operators. A reliable identification of the NP will involve the measurement of the NP parameters in many different ways, and with as many B decay modes as possible, so that it will be important to use all of these methods.

1 Introduction

Within the standard model (SM), CP violation is due to the presence of a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. The principal goal of the study of CP violation in the B system is to test this explanation, and to find evidence for physics beyond the SM. As a result, much theoretical work has concentrated on signals of new physics (NP) in B decays [2].

At present, we have several experimental hints of new physics. First, within the SM, the CP asymmetry in $B_d^0(t) \to J/\psi K_s$ should be approximately equal to that in decays dominated by the quark-level penguin transition $\bar{b} \to \bar{s}q\bar{q}$ ($q = u, d, s$). However, there is a 2.2σ difference between the Belle measurement of the CP asymmetry in $B_d^0(t) \to \phi K_s$ and that in $B_d^0(t) \to J/\psi K_s$ [3]. In addition, BaBar sees a 3.0σ discrepancy between the CP asymmetries measured in $B_d^0(t) \to \eta' K_s$ and $B_d^0(t) \to J/\psi K_s$ [4]. Second, the latest data on $B \to \pi K$ decays (branching ratios and various CP asymmetries) appear to be inconsistent with the SM [5]. Third, within the SM, one expects no triple-product asymmetries in $B \to \phi K^*$ [6], but BaBar has measured such an effect at 1.7σ [7].

It must be emphasized that none of these signals is statistically significant. Furthermore, the two experiments Belle and BaBar have not yet converged on any of the above measurements. Still, these signals are intriguing, particularly since the decays do share one thing in common: they all receive significant contributions from $\bar{b} \to \bar{s}$ penguin amplitudes. If NP is indeed present, it is therefore plausible to suspect that it is the $\bar{b} \to \bar{s}$ penguin which is principally affected. This is the assumption made in this paper.

All new-physics effects in B decays are necessarily virtual. Thus, regardless of how evidence for physics beyond the SM is found — any of the above hints could give a statistically-significant signal of NP with more data — many models can explain any discrepancy. As a result, it has generally been assumed that the *identification* of the NP will have to wait until the new particles are produced directly at high-energy colliders.

However, it was recently shown that this is not entirely true [8]. Briefly, the argument is as follows. Following the experimental hints, we assume that new physics contributes significantly to those decays which have large $\bar{b} \to \bar{s}$ penguin amplitudes. Consider now a $B \to f$ decay involving a $\bar{b} \to \bar{s}$ penguin. The NP operators are assumed to be roughly the same size as the SM $\bar{b} \to \bar{s}$ penguin operators, so the new effects are sizeable. At the quark level, the NP contributions take the form $\mathcal{O}_{NP}^{ij;q} \sim \bar{s}\Gamma_i b\,\bar{q}\Gamma_j q$ ($q = u, d, s, c$), where the $\Gamma_{i,j}$ represent Lorentz structures, and colour indices are suppressed. There are a total of 20 possible NP operators; each of them can in principle have a different weak phase.

There are new-physics contributions to the decay $B \to f$ through the matrix elements $\langle f | \mathcal{O}_{NP}^{ij;q} | B \rangle$. Each of these can be written as

$$\langle f | \mathcal{O}_{NP}^{ij;q} | B \rangle = A_k e^{i\phi_k^q} e^{i\delta_k^q} , \tag{1}$$

where ϕ_k^q and δ_k^q are the NP weak and strong phases associated with the individual matrix elements. However, it was argued in Ref. [8] that all NP strong phases are negligible compared to those of the SM. The point is that strong phases arise from rescattering. In the SM, this comes mainly from the tree diagram described at the quark level by $\bar{b} \to \bar{s}c\bar{c}$ [7]. However, note that this diagram is quite a bit larger than the $\bar{b} \to \bar{s}$ penguin diagrams. That is, the strong phases associated with penguin amplitudes are due to rescattering from a diagram which is considerably bigger. However, the NP strong phases can come only from rescattering from the NP diagrams themselves, which are much smaller than the SM tree diagram. Thus, the generated NP strong phases are correspondingly smaller than their SM counterparts. That is, the NP strong phases are negligible compared to the SM strong phases. (A detailed discussion of small NP strong phases is presented in the Appendix.)

Note that, in certain calculations of nonleptonic decays [9] it is claimed that the rescattering from the tree diagrams is negligible, but that annihilation terms, which are subleading ($\sim O(1/m_b)$), can be large. Large rescattering from the annihilation terms can generate a significant strong phase. If this scenario is true, then annihilation topologies associated with new-physics operators can also generate a large strong phase through rescattering. However, there is no general agreement on the importance of annihilation terms. Ultimately the size of annihilation diagrams is an experimental question, and can be tested by the measurement of decays such as $B_d^0 \to D_s^+ D_s^-$ and $B_d^0 \to K^+ K^-$. If the annihilations terms turn out to be small, as expected from the $O(1/m_b)$ suppression, then we can neglect the rescattering phase resulting from them. In our analysis we assume that annihilation-type topologies in the SM and with NP, which are power suppressed, are small, and therefore our argument that NP strong phases are negligible compared to those of the SM remains valid.

The observation that the NP strong phases are negligible allows for a great simplification: one can now combine all NP matrix elements into a single NP amplitude, with a single weak phase:

$$\sum \langle f | \, \mathcal{O}_{NP}^{ij,q} \, | B \rangle = \mathcal{A}^q e^{i\Phi_q} \; , \tag{2}$$

where $q = u, d, s, c$. Throughout the paper, we use the symbols $\mathcal{A}$ and Φ to denote the NP amplitudes and weak phases, respectively. In the above,

$$\tan \Phi_q \;\; = \;\; \frac{\sum_i A_i \sin \phi_i^q}{\sum_i A_i \cos \phi_i^q} \; . \tag{3}$$

Thus, all NP effects can be parametrized in terms of a small number of NP quantities. That is, we have an effective-lagrangian approach to new physics in CP violation in the B system.

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[7]There is also a $\bar{b} \to \bar{s}u\bar{u}$ tree diagram, but it is described by the product of CKM matrix elements $V_{ub}^* V_{us}$, which is tiny.

The argument the new-physics strong phases are negligible is quite general and applies to all NP models. However, this result can be obviated if special conditions are met. In particular, it does not hold if the NP is quite light, or if there is a significant enhancement of certain matrix elements. While these are disfavoured theoretically, the reader should be aware of these possible exceptions.

Note that, in general, $\mathcal{A}^q$ and Φ_q will be process-dependent. The NP phase Φ_q will be the same for all decays governed by the quark-level process $\bar{b} \to \bar{s}q\bar{q}$ only if all NP operators for the same quark-level process have the same weak phase. This is not uncommon. There are a number of NP models for which the weak phase is *universal* to all operators. These include models with Z- [10] and Z'-mediated [11] flavour-changing neutral currents (FCNC's), models in which the gluonic penguin operators have an enhanced chromomagnetic moment [12], and models with scalar-mediated FCNC's [13]. On the other hand, there are also NP models without universal weak phases, such as supersymmetric models with R-parity breaking, left-right symmetric models and models with four generations.

In Ref. [8], it was shown that the $\mathcal{A}^q$ and Φ_q can be *measured* using pairs of B decays which are related by flavour SU(3). One decay has a large $\bar{b} \to \bar{s}$ penguin component and so receives new-physics contributions. The other has a $\bar{b} \to \bar{d}$ penguin contribution. At present, there are no NP signals in processes which receive sizeable contributions from $\bar{b} \to \bar{d}$ penguin amplitudes, such as $B_d^0 \to \pi\pi$. In this technique, and in others like it, we therefore assume that the NP does not affect decays involving $\bar{b} \to \bar{d}$ penguins. The measurements of the two decays permit the extraction of the NP parameters, which in turn allows one to discriminate among NP models and rule out many of them. We can thus partially identify the new physics, before high-energy colliders are used.

In this paper, we provide a more detailed description of the method proposed in Ref. [8] to measure the NP parameters. We also examine two additional methods which can be used to obtain these NP parameters.

The first new method involves $B \to \pi K$ and $B \to \pi\pi$ decays. Recently, it was shown that, within the SM, the full unitarity triangle can be extracted from measurements of $B \to \pi K$ decays [14]. In order to do this, it is necessary to use flavour SU(3) to relate electroweak penguin operators to tree operators. On the other hand, if one assumes in addition the presence of new-physics $\bar{b} \to \bar{s}$ amplitudes in $B \to \pi K$, it is straightforward to show that there is not sufficient information to extract the various SM and NP parameters. However, flavour SU(3) also relates $B \to \pi K$ to $B \to \pi\pi$ decays. Since the NP is not expected to affect these latter decays, one can use flavour SU(3) to obtain certain SM $B \to \pi K$ amplitudes from $B \to \pi\pi$. With this information, it is possible to *measure* the NP parameters. The advantage of this method is that the analysis can be performed with present data; the disadvantage is that there is a theoretical error due to the assumption of flavour SU(3) symmetry.

The second new method involves $B \to V_1 V_2$ decays, where V_1 and V_2 are vector

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mesons. These decays are very promising for finding evidence of physics beyond the SM. Suppose that the final state is such that (i) $\overline{V}_1\overline{V}_2 = V_1V_2$, and (ii) a single decay amplitude dominates in the SM. In this case, a time-dependent angular analysis of $B(t) \to V_1V_2$ provides numerous signals of new physics [15]. Suppose further that a single NP amplitude is present, with a different weak phase from that of the SM amplitude and a (helicity-dependent) strong phase. The NP weak phase is assumed to be helicity-independent, which is the case for NP models whose weak phase is universal to all operators. In Ref. [15] it was shown that one can place lower bounds on the NP parameters. However, we have argued above that the NP strong phase is negligible, in which case the analysis is modified. As we will see, there are now more observables than theoretical parameters, so that one can *measure* the NP parameters in this system. Compared to Ref. [8], the advantage is that no theoretical input [flavour SU(3)] is required; the disadvantage is that it is difficult experimentally, and the analysis only holds for a certain class of NP models.

We begin in Sec. 2 with a detailed discussion of the method involving $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ penguin decays which are related by SU(3). In Sec. 3 we turn to the analysis of $B \to \pi K$ and $B \to \pi\pi$ decays. The technique involving $B \to V_1V_2$ decays is examined in Sec. 4. In Sec. 5 we discuss all three methods, stressing their advantages and disadvantages. All methods have their own unique features, and a complete analysis would ideally include all three techniques. We also examine two models of new physics, and show that different NP models lead to different patterns of NP parameters. This demonstrates that the measurement of the NP parameters does indeed discriminate among various models, and provides a partial identification of the NP. We conclude in Sec. 6.

2 B Penguin Decays

We begin with a description of the method proposed in Ref. [8] for measuring new-physics parameters. This technique closely resembles that of Ref. [16], which two of us (AD, DL) recently proposed for extracting CP phase information. Here the method is reversed. We assume that NP is present only in decays with large $\bar{b} \to \bar{s}$ penguin amplitudes, and that the SM CP phase information is known (the SM phases can be measured using processes which do not involve large $\bar{b} \to \bar{s}$ penguin amplitudes). We first study the general case in abstract terms. We then apply the method to specific decays.

2.1 General Case

We begin by considering a neutral $B'^0 \to M'_1M'_2$ decay in which the final state $M'_1M'_2$ is accessible to both B^0 and $\overline{B}^0$ mesons. B'^0 can be either a B_d^0 or a B_s^0 meson, and M'_1 and M'_2 are two mesons. (If both M'_1 and M'_2 are vector mesons, the final state can be considered as a single helicity state of $M'_1M'_2$.) This decay involves a $\bar{b} \to \bar{s}$

penguin contribution and is dominated by a single decay amplitude in the SM. (The case in which there are two significant SM amplitudes is discussed at the end of this subsection.)

Since $B'^0 \to M_1' M_2'$ involves a $\bar{b} \to \bar{s}$ penguin amplitude, new physics is present. As discussed in the introduction, because the NP strong phases are negligible, the effect of the NP can be parametrized in terms of a single effective amplitude, with a NP weak phase. Thus, including the NP, the amplitude for $B'^0 \to M_1' M_2'$ can be written

$$A(B'^0 \to M_1' M_2') \equiv A = \mathcal{A}'_{ct}\, e^{i\delta'_{ct}} + \mathcal{A}^q e^{i\Phi_q} \ , \tag{4}$$

where $\mathcal{A}'_{ct}$ and $\mathcal{A}^q$ are the SM and NP amplitudes, respectively. Similarly, δ'_{ct} and Φ_q are the SM strong phase and NP weak phase, respectively. The amplitude for the CP-conjugate process, $\overline{A}$, can be obtained from the above by changing the sign of the weak phase Φ_q.

The time-dependent measurement of $B'^0(t) \to M_1' M_2'$ allows one to obtain the three observables

$$
\begin{aligned}
B &\equiv \frac{1}{2}\left(|A|^2 + |\overline{A}|^2\right) = (\mathcal{A}'_{ct})^2 + (\mathcal{A}^q)^2 + 2\mathcal{A}'_{ct}\,\mathcal{A}^q \cos\delta'_{ct} \cos\Phi_q \ , \\
a_{dir} &\equiv \frac{1}{2}\left(|A|^2 - |\overline{A}|^2\right) = 2\mathcal{A}'_{ct}\,\mathcal{A}^q \sin\delta'_{ct} \sin\Phi_q \ , \\
a_I &\equiv \mathrm{Im}\left(e^{-2i\phi_M^q} A^* \overline{A}\right) = -(\mathcal{A}'_{ct})^2 \sin 2\phi_M^q - 2\mathcal{A}'_{ct}\,\mathcal{A}^q \cos\delta'_{ct} \sin(2\phi_M^q + \Phi_q) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad -(\mathcal{A}^q)^2 \sin(2\phi_M^q + 2\Phi_q) \ .
\end{aligned}
\tag{5}
$$

It is useful to define a fourth observable:

$$
\begin{aligned}
a_R &\equiv \mathrm{Re}\left(e^{-2i\phi_M^q} A^* \overline{A}\right) = (\mathcal{A}'_{ct})^2 \cos 2\phi_M^q + 2\mathcal{A}'_{ct}\,\mathcal{A}^q \cos\delta'_{ct} \cos(2\phi_M^q + \Phi_q) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad +(\mathcal{A}^q)^2 \cos(2\phi_M^q + 2\Phi_q) \ .
\end{aligned}
\tag{6}
$$

The quantity a_R is not independent of the other three observables:

$$a_R{}^2 = B^2 - a_{dir}{}^2 - a_I{}^2 \ . \tag{7}$$

Thus, one can obtain a_R from measurements of B, a_{dir} and a_I, up to a sign ambiguity.

In the above, ϕ_M^q $(q = d, s)$ is the phase of B'^0–$\bar{B}'^0$ mixing. For $B'^0 = B_d^0$, this phase is unaffected by new physics and thus takes its SM value, β. The canonical way to measure this angle is via CP violation in $B_d^0(t) \to J/\psi K_S$. However, there is a potential problem here: this decay receives NP contributions from $O_{NP}^c \sim \bar{s}b\bar{c}c$ operators (the Lorentz and colour structures have been suppressed). On the other hand, the value of β extracted from $B_d^0(t) \to J/\psi K_S$ is in line with SM expectations. This strongly suggests that any O_{NP}^c contributions to this decay are quite small.

The situation is somewhat different for $B'^0 = B_s^0$. In general, NP which affects $\bar{b} \to \bar{s}$ transitions will also contribute to B_s^0–$\overline{B_s^0}$ mixing, i.e. one will have NP operators of the form $\bar{s}b\bar{s}b$. In this case, the phase of B_s^0–$\overline{B_s^0}$ mixing may

well differ from its SM value ($\simeq 0$) due to the presence of NP. The standard way to measure this mixing phase is through CP violation in $B_s^0(t) \to J/\psi\eta$ (or $B_s^0(t) \to J/\psi\phi$). As with $B_d^0(t) \to J/\psi K_S$, these decays potentially receive O_{NP}^c contributions. However, since the non-strange part of the η wavefunction has a negligible contribution to $\langle J/\psi\eta| O_{NP}^c |B_s^0\rangle$, this matrix element can be related by flavour SU(3) to $\langle J/\psi K_S| O_{NP}^c |B_d^0\rangle$ (up to a mixing angle). That is, both matrix elements are very small. In other words, we do not expect significant O_{NP}^c contributions to $B_s^0(t) \to J/\psi\eta$, and the phase of B_s^0–$\overline{B_s^0}$ mixing can be measured through CP violation in this decay, even in the presence of NP.

Another decay which can be used to measure the phase of B_s^0–$\overline{B_s^0}$ mixing is $B_s^0(t) \to D_s^+ D_s^-$. Since the final state $D_s^+ D_s^-$ is unrelated to $J/\psi\eta$, it is logically possible that O_{NP}^c will have measurable effects in $B_s^0(t) \to D_s^+ D_s^-$. The easiest way to detect this is to measure the B_s^0–$\overline{B_s^0}$ mixing phase in both $B_s^0(t) \to J/\psi\eta$ and $B_s^0(t) \to D_s^+ D_s^-$. If these two phases differ, this will clearly signal the presence of NP in $\bar{b} \to \bar{s}c\bar{c}$ transitions. However, such measurements will not allow us to cleanly determine the magnitude and phase of O_{NP}^c. This can be done using the technique described in this section.

Note that the expressions for a_{dir} and a_I in Eq. (5) provide several clear signals of NP. Since $B'^0 \to M_1' M_2'$ is dominated by a single decay in the SM, the direct CP asymmetry is predicted to vanish. Furthermore, in the SM the indirect CP asymmetry measures the mixing phase ϕ_M^q. Thus, if it is found that $a_{dir} \neq 0$, or that ϕ_M^q differs from its SM value, this would be a smoking-gun signal of NP. Note also that, if it happens that the SM strong phases are small, a_{dir} may be unmeasurable. In this case, a better signal of new physics is the measurement of T-violating triple-product correlations in the corresponding vector-vector final states [6]. This is an example of the many NP signals present in B decays. However, these signals do not, by themselves, allow the measurement of the NP parameters.

The three independent observables of Eqs. (5) and (6) depend on four unknown theoretical parameters: $\mathcal{A}^q$, $\mathcal{A}_{ct}'$, δ_{ct}' and Φ_q. Therefore one cannot obtain information about the new-physics parameters $\mathcal{A}^q$ and Φ_q from these measurements. However, one can partially solve the equations to obtain

$$(\mathcal{A}_{ct}')^2 = \frac{a_R \cos(2\phi_M^q + 2\Phi_q) - a_I \sin(2\phi_M^q + 2\Phi_q) - B}{\cos 2\Phi_q - 1} \ . \qquad (8)$$

From this expression, we see that, if we knew $\mathcal{A}_{ct}'$, we could solve for Φ_q.

In order to get $\mathcal{A}_{ct}'$ we consider the partner process $B^0 \to M_1 M_2$ involving a $\bar{b} \to \bar{d}$ penguin amplitude. In the SM this decay is related by SU(3) symmetry to $B'^0 \to M_1' M_2'$. (In some cases, this relation only holds if one neglects annihilation- or exchange-type diagrams [16], which are expected to be small.) B^0 can be either a B_d^0 or a B_s^0 meson and, as with $B'^0 \to M_1' M_2'$, it is assumed that both B^0 and $\overline{B}^0$ can decay to the final state $M_1 M_2$. The partner process can be a pure penguin decay, or can involve both tree and (non-negligible) penguin contributions.

Since $\bar{b} \to \bar{s}$ transitions are not involved, the amplitude for $B^0 \to M_1 M_2$ receives only SM contributions, and is given by

$$
\begin{aligned}
A(B^0 \to M_1 M_2) &= A_u V_{ub}^* V_{ud} + A_c V_{cb}^* V_{cd} + A_t V_{tb}^* V_{td} \\
&= (A_u - A_t) V_{ub}^* V_{ud} + (A_c - A_t) V_{cb}^* V_{cd} \\
&\equiv \mathcal{A}_{ut}\, e^{i\gamma} e^{i\delta_{ut}} + \mathcal{A}_{ct}\, e^{i\delta_{ct}} ,
\end{aligned}
\tag{9}
$$

where $\mathcal{A}_{ut} \equiv |(A_u - A_t) V_{ub}^* V_{ud}|$, $\mathcal{A}_{ct} \equiv |(A_c - A_t) V_{cb}^* V_{cd}|$, and we have explicitly written the strong phases δ_{ut} and δ_{ct}, as well as the weak phase γ. In the above, we adopt the c-quark convention [17], in which CKM unitarity is used to eliminate the t-quark term.

As with $B'^0 \to M_1' M_2'$, the time-dependent measurement of $B^0(t) \to M_1 M_2$ allows one to obtain three independent observables [Eqs. (5) and (6)]. These observables depend on five theoretical quantities: $\mathcal{A}_{ct}$, $\mathcal{A}_{ut}$, $\delta \equiv \delta_{ut} - \delta_{ct}$, γ and the mixing phase ϕ_M^q. However, as discussed above, ϕ_M^q can be measured independently using processes which are unaffected by new physics in $\bar{b} \to \bar{s}$ penguin amplitudes. The weak phase γ can be measured similarly. For example, it can be obtained from $B^\pm \to DK$ decays [18]. Alternatively, the angle α can be extracted from $B \to \pi\pi$ [19], $B \to \rho\pi$ [20] or $B \to \rho\rho$ decays [21], and γ can be obtained using $\gamma = \pi - \beta - \gamma$. Given that these CP phases can be measured independently, the three observables of $B^0(t) \to M_1 M_2$ now depend on three unknown theoretical parameters, so that the system of equations can be solved.

In particular, one can obtain $\mathcal{A}_{ct}$:

$$
\mathcal{A}_{ct}^2 = \frac{a_R \cos(2\phi_M^q + 2\gamma) - a_I \sin(2\phi_M + 2\gamma) - B}{\cos 2\gamma - 1} ,
\tag{10}
$$

where a_R, a_I and B are the observables found in $B^0(t) \to M_1 M_2$.

The key point is that, in the SU(3) limit, one has

$$
\mathcal{A}_{ct} = \lambda \mathcal{A}_{ct}' ,
\tag{11}
$$

where $\lambda = 0.22$ is the Cabibbo angle. With this relation, the extraction of $\mathcal{A}_{ct}$ from $B^0(t) \to M_1 M_2$ yields $\mathcal{A}_{ct}'$. Thus, Eq. (8) can be used to solve for the new physics phase Φ_q. The NP amplitude $\mathcal{A}^q$ can also be obtained. There is a theoretical error in the above relation due to SU(3)-breaking effects. However, various methods were discussed in Ref. [16] to reduce this SU(3) breaking. All of these methods are applicable here. In the end, depending on which pair of processes is used, the theoretical error can be reduced to the level of 5–15%.

At this point we can make an important general observation. As noted in the introduction [Eq. (3)], the new-physics weak phase Φ_q depends on the matrix elements of the various NP operators for the particular process considered (A_i), as well as the corresponding weak phases ϕ_i^q. Thus, in general, the value of Φ_q extracted from two distinct decay pairs with the same underlying $\bar{b} \to \bar{s}q\bar{q}$ transition will be

different. There are two reasons for this. First, certain operators which contribute to one process may not contribute in the same form to another. (For example, one decay might be colour-suppressed, while the other is colour-allowed.) Second, even if the same operators are involved (with the same form) in two $\bar{b} \to \bar{s}q\bar{q}$ decays, the matrix elements of the various operators will depend on the final states considered. Thus, the A_i in Eq. (3) are process-dependent in general, and the value of the phase Φ_q depends on the particular decay pair used. However, if all NP operators for the quark-level process $\bar{b} \to \bar{s}q\bar{q}$ have the same weak phase ϕ^q, then the NP phase Φ_q will be the same for all decays governed by the same quark-level process. Hence it is important to measure the phase Φ_q in more than one pair of processes with the same underlying quark transition. If the effective phases are different then it would be a clear signal of more than one NP amplitude, with different weak phases, in $\bar{b} \to \bar{s}q\bar{q}$. Furthermore, in some NP models, the phases for the different underlying quark transitions $\bar{b} \to \bar{s}q\bar{q}$ are related, so that the NP phase is independent of the quark flavour.

In the above method, we have assumed that the decay $B'^0 \to M_1' M_2'$ is dominated by a single decay amplitude in the SM. This is the case only for the quark-level decays $\bar{b} \to \bar{s}q\bar{q}$ ($q = d, s, c$). However, it is straightforward to adapt this technique to $\bar{b} \to \bar{s}u\bar{u}$, for which $B'^0 \to M_1' M_2'$ receives both tree and $\bar{b} \to \bar{s}$ penguin contributions in the SM. The process $B_s^0 \to K^+ K^-$ is an example of such a decay. Including the new-physics contribution, the amplitude for such decays can be written

$$A(B'^0 \to M_1' M_2') = \mathcal{A}_{ut}' \; e^{i\gamma} e^{i\delta_{ut}'} + \mathcal{A}_{ct}' \; e^{i\delta_{ct}'} + \mathcal{A}^u e^{i\Phi_u} \; . \tag{12}$$

Here, assuming that γ and the mixing phase ϕ_M^q are known, the three independent observables in this decay depend on six unknown parameters: $\mathcal{A}_{ct}'$, $\mathcal{A}_{ut}'$, δ_{ut}', δ_{ct}', $\mathcal{A}^u$ and Φ_u. In this case, in order to solve for the NP parameters, one needs three pieces of information. These can be obtained as follows. Measurements of the partner process allow one to extract $\mathcal{A}_{ct}$, $\mathcal{A}_{ut}$ and $\delta \equiv \delta_{ut} - \delta_{ct}$. We now assume that

$$\mathcal{A}_{ct} = \lambda \mathcal{A}_{ct}' \; , \quad \lambda \mathcal{A}_{ut} = \mathcal{A}_{ut}' \; , \quad \delta' = \delta \; , \tag{13}$$

where $\delta' \equiv \delta_{ut}' - \delta_{ct}'$. These assumptions then permit the extraction of $\mathcal{A}^u$ and Φ_u from measurements of $B'^0 \to M_1' M_2'$.

However, the theoretical uncertainty here due to SU(3) breaking is considerably larger than in the case where $B'^0 \to M_1' M_2'$ is dominated by a single amplitude in the SM. Not only do we relate two amplitudes instead of one [Eq. (11)], but we also assume that two strong phases are equal. Thus, the NP parameters $\mathcal{A}^u$ and Φ_u can be obtained in this way, but we expect a larger theoretical error.

2.2 Specific Decays

In Ref. [16], we showed that there are twelve decay pairs $B^0 \to M_1 M_2$ and $B' \to M_1' M_2'$ which can be used to obtain CP phase information in the SM, with a small

NP Parameters	$B'^0(t) \to M_1' M_2'$	$B^0(t) \to M_1 M_2$
$\Phi_c, \mathcal{A}^c$	$B_s^0(t) \to D_s^+ D_s^-$	$B_d^0(t) \to D^+ D^-$
$\Phi_s, \mathcal{A}^s$	$B_d^0(t) \to \phi K^{*0}$ $B_s^0(t) \to \phi\phi$	$B_s^0(t) \to \phi \bar{K}^{*0}$ $B_s^0(t) \to \phi \bar{K}^{*0}$
$\Phi_d, \mathcal{A}^d$	$B_s^0(t) \to K^0 \bar{K}^0$ $B_s^0(t) \to K^0 \bar{K}^0$ $B_d^0(t) \to K^{*0} \rho^0$ $B_d^0(t) \to K^{*0} \rho^0$	$B_d^0(t) \to \pi^+ \pi^-$ $B_d^0(t) \to K^0 \bar{K}^0$ $B_d^0(t) \to \rho^0 \rho^0$ $B_s^0(t) \to \bar{K}^{*0} \rho^0$
$\Phi_u, \mathcal{A}^u$	$B_s^0(t) \to K^+ K^-$	$B_d^0(t) \to \pi^+ \pi^-$

Table 1: For each set of new-physics parameters $\mathcal{A}^q$ and Φ_q, we list the B decays $(B'^0(t) \to M_1' M_2')$ and their partner processes $(B^0(t) \to M_1 M_2)$ which can be used to measure them.

theoretical error. (In fact, there are more, since many of the particles in the final states can be observed as either pseudoscalar (P) or vector (V) mesons.) Many of these decay pairs can also be used to measure the new-physics parameters, assuming that the NP contributes significantly only to the $\bar{b} \to \bar{s}$ decays, and that the SM CP phases have already been measured using non-$\bar{b} \to \bar{s}$ processes.

As noted earlier, assuming that new-physics strong rescattering is negligible relative to that of the SM, all NP effects can be parametrized in terms of the effective NP amplitudes $\mathcal{A}^q$ and weak phases Φ_q ($q = u, d, s, c$), independent of the type of underlying NP [8]. Of the above twelve decay pairs, seven involve only neutral B-mesons and have final states $M_1' M_2'$ accessible to both B^0 and $\overline{B}^0$. These can be used to measure the NP parameters $\mathcal{A}^q$ and Φ_q ($q = d, s, c$) using the method outlined in Sec. 2.1. For $\mathcal{A}^u$ and Φ_u, we have to use a decay pair in which $B' \to M_1' M_2'$ receives both tree and penguin contributions in the SM. There is one such possibility. Thus, all four sets of NP parameters can be obtained from measurements of pairs of B decays.

The decay pairs are listed in Table 1, along with the new-physics parameters probed. We have several comments about these.

There are three reasons why certain decays are written in terms of vector-vector (VV) final states, while others involve pseudoscalar-pseudoscalar (PP) states. First, some decays involve a final-state π^0. However, experimentally it will be necessary to find the decay vertices of the final particles. This is virtually impossible for a π^0, and so we always use a ρ^0 [22]. Second, some pairs of decays are related by SU(3) in the SM only if an ($s\bar{s}$) quark pair is used. Unfortunately, there are no P's which are pure ($s\bar{s}$). The mesons η and η' have an ($s\bar{s}$) component, but they also have significant ($u\bar{u}$) and ($d\bar{d}$) pieces. As a result the decays $B' \to M_1' M_2'$ and $B^0 \to M_1 M_2$ are not really related by SU(3) in the SM if the final state involves

an η or η'. We therefore consider instead the vector meson ϕ which is essentially a pure $(s\bar{s})$ quark state. Finally, we require that both B^0 and $\bar{B}^0$ be able to decay to the final state. This cannot happen if the final state contains a single K^0 (or $\bar{K}^0$) meson. However, it can occur if this final-state particle is an excited neutral kaon. In this case one decay involves K^{*0}, while the other has $\bar{K}^{*0}$. Assuming that the vector meson is detected via its decay to $K_s\pi^0$ (as in the measurement of $\sin 2\beta$ via $B_d^0(t) \to J/\psi K^*$), then both B^0 and $\bar{B}^0$ can decay to the same final state.

Apart from these three restrictions, the final-state particles can be taken to be either pseudoscalar or vector. Indeed, it will be useful to measure the NP parameters in modes with PP, PV and VV final-state particles, since different NP operators are probed in these decays. For example, within factorization, certain scalar operators cannot contribute to PV or VV states if their amplitudes involve the matrix element $\langle V | \bar{q}\gamma_{L,R}q | 0 \rangle$. In general, the matrix element of a given operator will be different for the various PP, PV and VV final states. Thus, the measurement of the NP parameters in different modes will provide some clues as to which NP operators are present.

In addition, if it is found that Φ_q is different for decays governed by the same underlying quark-level transition, it will indicate the presence of more than one NP amplitude, with different weak phases.

For the NP parameters $\mathcal{A}^q$ and Φ_q $(q = d, s, c)$, the theoretical error is due to SU(3) breaking in Eq. (11), and can be reduced to the range 5–15% [16]. On the other hand, the only way to measure Φ_u and $\mathcal{A}^u$ is to use $B_s^0(t) \to K^+K^-$ and $B_d^0(t) \to \pi^+\pi^-$. However, $B_s^0 \to K^+K^-$ has both tree and penguin contributions. In order to obtain Φ_u and $\mathcal{A}^u$, it is therefore necessary to make the three assumptions in Eq. (13). In the context of measuring the angle γ, the SU(3) breaking in $B_s^0 \to K^+K^-$ and $B_d^0 \to \pi^+\pi^-$ was examined in Refs. [23, 24]. In the framework of naive factorization or QCD factorization[25], it can be shown that, as long as annihilation-type topologies are small, the double ratio of amplitudes $\lambda^2(\mathcal{A}'_{ct}/\mathcal{A}'_{ut})/(\mathcal{A}_{ct}/\mathcal{A}_{ut})$ has small SU(3) breaking. However, Eq. (13) does not involve this double ratio of amplitudes – it involves single amplitude ratios (and an equality of strong phases). In this case, even within QCD factorization with small annihilation-type topologies, there are several sources of SU(3) breaking which are not under total control. The SU(3) breaking comes from the difference between unknown $B_s^0 \to K$ and $B_d^0 \to \pi$ form factors, differences in the light cone distributions of the kaon and the pion, and other subleading but potentially important unknown soft physics [26]. As a result, putting all these SU(3)-breaking effects together, the theoretical error in the extraction of Φ_u and $\mathcal{A}^u$ is quite a bit larger than for the measurement of the other NP parameters.

Note that only one pair in Table 1 involves only B_d^0 decays. The others will require the time-dependent measurement of B_s^0 decays. However, this will be difficult experimentally, as B_s^0–$\overline{B_s^0}$ mixing is large. For this reason the decay pair $B_d^0(t) \to K^{*0}\rho^0$ and $B_d^0(t) \to \rho^0\rho^0$ may be the most promising for measuring NP parameters

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using this method.

3 $B \to \pi K$ and $B \to \pi\pi$ Decays

In this section we consider $B \to \pi K$ and $B \to \pi\pi$ decays. It is well known that it is possible to express the amplitudes for B decays to two pseudoscalars in terms of a number of distinct SU(3) operators. This is equivalent to a description in terms of diagrams [27]. Neglecting the exchange- and annihilation-type diagrams, which are expected to be small for dynamical reasons, but including electroweak penguin contributions (EWP's), there are five diagrams [28]: (1) a colour-favored tree amplitude T (or T'), (2) a colour-suppressed tree amplitude C (or C'), (3) a gluonic penguin amplitude P (or P'), (4) a colour-favored electroweak penguin amplitude P_{EW} (or P'_{EW}), and (5) a colour-suppressed electroweak penguin amplitude P^C_{EW} (or P'^C_{EW}). In the following, we denote all diagrams contributing to $\bar{b} \to \bar{d}$ ($\bar{b} \to \bar{s}$) decays without (with) primes.

As described in Sec. 2, the penguin diagram actually contains several pieces:

$$\begin{aligned} P &= P_u V^*_{ub}V_{ud} + P_c V^*_{cb}V_{cd} + P_t V^*_{tb}V_{td} \\ &= (P_u - P_t)V^*_{ub}V_{ud} + (P_c - P_t)V^*_{cb}V_{cd} \\ &\equiv P_{ut}\, e^{i\gamma} + P_{ct} \; . \end{aligned} \tag{14}$$

In the above, we have explicitly written the weak phase γ; the amplitudes implicitly include the strong phases. As in Sec. 2, we have adopted the c-quark convention [17] in passing from the first line to the second. For $\bar{b} \to \bar{s}$ decays, we can write P' analogously to the above, except that we expect $|P'_{ut}| \ll |P'_{ct}|$ since $|V^*_{ub}V_{us}/V^*_{cb}V_{cs}| \simeq$ 2%. In Sec. 2 we neglected the P'_{ut} term. We will eventually do something similar here as well, but for the moment we keep all terms in the $B \to \pi K$ amplitudes.

For $\bar{b} \to \bar{d}$ decays, the EWP contributions are expected to be negligible. However, they are important for $\bar{b} \to \bar{s}$ transitions [28]. It was recently shown that, to a good approximation, the EWP's can be related to tree operators using Fierz transformations and SU(3) symmetry [29]. Ignoring exchange- and annihilation-type diagrams once again, the relations are

$$\begin{aligned} P'_{EW} &= \frac{3}{4}\frac{c_9 + c_{10}}{c_1 + c_2}R(T' + C') + \frac{3}{4}\frac{c_9 - c_{10}}{c_1 - c_2}R(T' - C') \; , \\ P'^C_{EW} &= \frac{3}{4}\frac{c_9 + c_{10}}{c_1 + c_2}R(T' + C') - \frac{3}{4}\frac{c_9 - c_{10}}{c_1 - c_2}R(T' - C') \; , \end{aligned} \tag{15}$$

where the c_i are Wilson coefficients. Here, the weak phases have been factored out, so these relations include only strong phases. In the above,

$$R \equiv \left| \frac{V^*_{tb}V_{ts}}{V^*_{ub}V_{us}} \right| = \frac{1}{\lambda^2\sqrt{\rho^2 + \eta^2}} \; , \tag{16}$$

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where ρ and η are the CKM parameters. (Note that the three CP phases in the unitarity triangle are functions of ρ and η.) Thus, all $B \to PP$ amplitudes can be expressed in terms of T, C, P_{ut}, P_{ct} (or their $\bar{b} \to \bar{s}$ equivalents) and the weak phases.

We now turn to the four $B \to \pi K$ decays $B^+ \to \pi^+ K^0$, $B^+ \to \pi^0 K^+$, $B_d^0 \to \pi^- K^+$ and $B_d^0 \to \pi^0 K^0$. There are 9 measurements that can be made of this system: four branching ratios, four direct asymmetries, and one indirect asymmetry (in $B_d^0(t) \to \pi^0 K_s$). However, assuming that the P_{ut}' term is negligible, in the SM the amplitudes can be expressed in terms of 7 parameters: three diagram magnitudes, two relative strong phases, and two CKM parameters. Thus there is enough information in $B \to \pi K$ decays to reconstruct the full unitarity triangle [14].

We now consider new physics. $B \to \pi K$ decays are $\bar{b} \to \bar{s}$ transitions, so that NP can affect these decays. There are two classes of NP operators, differing in their colour structure: $\bar{s}_\alpha \Gamma_i b_\alpha \, \bar{q}_\beta \Gamma_j q_\beta$ and $\bar{s}_\alpha \Gamma_i b_\beta \, \bar{q}_\beta \Gamma_j q_\alpha$. The first class of NP operators contributes with no colour suppression to final states containing $\bar{q}q$ mesons. The second type of operator can also contribute via Fierz transformations, but there is a suppression factor of $1/N_c$, as well as additional operators involving colour octet currents. Similarly, for final states with $\bar{s}q$ mesons, the roles of the two classes of operators are reversed. We denote by $\mathcal{A}'^{,q} e^{i\Phi_q'}$ and $\mathcal{A}'^{C,q} e^{i\Phi_q'^C}$ the sum of NP operators which contribute to final states involving $\bar{q}q$ and $\bar{s}q$ mesons, respectively. Here, Φ_q' and $\Phi_q'^C$ are the NP weak phases; the strong phases are zero. We stress that, despite the "colour-suppressed" index C, the operators $\mathcal{A}'^{C,q} e^{i\Phi_q'^C}$ are not necessarily smaller than the $\mathcal{A}'^{,q} e^{i\Phi_q'}$.

Including these NP operators, the $B \to \pi K$ amplitudes can be written

$$A(B^+ \to \pi^+ K^0) = P_{ut}' e^{i\gamma} + P_{ct}' - \frac{1}{3} P_{EW}'^C + \mathcal{A}'^{C,d} e^{i\Phi_d'^C} \ ,$$

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = -P_{ut}' e^{i\gamma} - P_{ct}' - T' e^{i\gamma} - C' e^{i\gamma} - P_{EW}'$$
$$- \frac{2}{3} P_{EW}'^C - \mathcal{A}'^{,u} e^{i\Phi_u'} + \mathcal{A}'^{,d} e^{i\Phi_d'} - \mathcal{A}'^{C,u} e^{i\Phi_u'^C} \ ,$$

$$A(B_d^0 \to \pi^- K^+) = -P_{ut}' e^{i\gamma} - P_{ct}' - T' e^{i\gamma} - \frac{2}{3} P_{EW}'^C - \mathcal{A}'^{C,u} e^{i\Phi_u'^C} \ ,$$

$$\sqrt{2} A(B_d^0 \to \pi^0 K^0) = P_{ut}' e^{i\gamma} + P_{ct}' - C' e^{i\gamma} - P_{EW}' - \frac{1}{3} P_{EW}'^C$$
$$- \mathcal{A}'^{,u} e^{i\Phi_u'} + \mathcal{A}'^{,d} e^{i\Phi_d'} + \mathcal{A}'^{C,d} e^{i\Phi_d'^C} \ . \qquad (17)$$

Here, each of P_{ut}', P_{ct}', T' and C' include a (different) strong phase. Note that $\mathcal{A}'^{,u} e^{i\Phi_u'}$ and $\mathcal{A}'^{,d} e^{i\Phi_d'}$ always appear in the same combination above. We therefore define $\mathcal{A}'^{,comb} e^{i\Phi'} \equiv -\mathcal{A}'^{,u} e^{i\Phi_u'} + \mathcal{A}'^{,d} e^{i\Phi_d'}$. It is not possible to distinguish the two component amplitudes.

In the presence of NP, the amplitudes can be written in terms of 16 theoretical quantities: 7 amplitude magnitudes ($|P_{ut}'|$, $|P_{ct}'|$, $|T'|$, $|C'|$, $|\mathcal{A}'^{,comb}|$, $|\mathcal{A}'^{C,d}|$ and $|\mathcal{A}'^{C,u}|$), 4 relative strong phases, 2 SM weak phases, and 3 NP weak phases. (In

the following, we generically refer to the strong phases, SM weak phases and NP weak phases as "δ", "ϕ" and "Φ," respectively.) Since we have 16 parameters and only 9 measurements, it is clear that we cannot measure the NP parameters using $B \to \pi K$ alone. It does not help to make the approximation that P'_{ut} is negligible.

Important information can be obtained from measurements of the $B \to \pi\pi$ system. As per our assumptions, new physics does not affect such decays. Neglecting EWP contributions, which are expected to be small, the $B \to \pi\pi$ amplitudes can be written

$$
\begin{aligned}
\sqrt{2} A(B^+ \to \pi^+ \pi^0) &= -T\, e^{i\gamma} - C\, e^{i\gamma} \ , \\
A(B^0_d \to \pi^+ \pi^-) &= -T\, e^{i\gamma} - P_{ut}\, e^{i\gamma} - P_{ct} \ , \\
\sqrt{2} A(B^0_d \to \pi^0 \pi^0) &= -C\, e^{i\gamma} + P_{ut}\, e^{i\gamma} + P_{ct} \ .
\end{aligned}
\tag{18}
$$

The indirect asymmetry in $B^0_d(t) \to \pi^+ \pi^-$ also involves the phase of B^0_d–$\overline{B^0_d}$ mixing, β, so the 6 $B \to \pi\pi$ measurements [three branching ratios, two direct asymmetries, and one indirect asymmetry (in $B^0_d(t) \to \pi^+ \pi^-$)] are a function of 7 theoretical parameters. However, if one assumes that β is measured in $B^0_d(t) \to J/\psi K_S$, all theoretical quantities can be extracted [30].

Within flavour SU(3) symmetry, the $B \to \pi\pi$ amplitudes are related to those in $B \to \pi K$:

$$
\frac{T'}{T} = \frac{C'}{C} = \frac{P'_{ut}}{P_{ut}} = \frac{\lambda}{1 - \lambda^2/2} \ , \quad \frac{P_{ct}}{P'_{ct}} = \frac{P_{EW}}{P'_{EW}} = \frac{P^C_{EW}}{P'^C_{EW}} = -\frac{\lambda}{1 - \lambda^2/2} \ ,
\tag{19}
$$

where the various amplitudes include the strong phases. With these relations, we can combine the information obtained in $B \to \pi\pi$ and $B \to \pi K$ decays. As detailed above, there are a total of 16 theoretical parameters. However, there are now also 16 experimental measurements: 6 in $B \to \pi\pi$, 9 in $B \to \pi K$ and the extraction of β in $B^0_d(t) \to J/\psi K_s$. It is therefore in principle possible to solve for all theoretical unknowns, and we would thus *measure* the NP parameters.

Unfortunately, solving 16 nonlinear equations in 16 unknowns will lead to a large number of discretely-ambiguous solutions. When one adds the experimental errors, the solutions will be smeared out, and the values of the NP parameters will essentially remain unknown. To remedy this, we adopt the procedure of Ref. [31] in order to reduce the number of theoretical unknowns. First, we neglect the P'_{ut} term in the decays $B^+ \to \pi^+ K^0$ and $B^+ \to \pi^0 K^+$. Second, we remove the dependence on P_{ut} by redefining T and C:

$$
\tilde{T} = T + P_{ut} \ , \quad \tilde{C} = C - P_{ut} \ ,
\tag{20}
$$

with similar redefinitions for the primed quantities. Finally, the relations in Eq. (15) no longer hold when $\tilde{T}'$ and $\tilde{C}'$ are used. We therefore neglect the amplitude P'^C_{EW}. With this, there is a single relation between P'_{EW} and the tree diagrams:

$$
P'_{EW} = \frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} R(\tilde{T}' + \tilde{C}') \ .
\tag{21}
$$

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Since $|P'_{ut}|, |P'^C_{EW}| \ll |P'_{ct}|$ we expect the error associated with these approximations to be small. Indeed, in Ref. [31], SM fits both with and without the approximations were performed, and little difference was found.

With the above approximations, the $B \to \pi K$ amplitudes take the form

$$
\begin{aligned}
A(B^+ \to \pi^+ K^0) &= P'_{ct} + \mathcal{A}'^{C,d} e^{i\Phi'^C_d} \ , \\
\sqrt{2} A(B^+ \to \pi^0 K^+) &= -P'_{ct} - \tilde{T}' \, e^{i\gamma} - \tilde{C}' \, e^{i\gamma} - P'_{EW} + \mathcal{A}'^{,comb} e^{i\Phi'} - \mathcal{A}'^{C,u} e^{i\Phi'^C_u} \ , \\
A(B^0_d \to \pi^- K^+) &= -P'_{ct} - \tilde{T}' \, e^{i\gamma} - \mathcal{A}'^{C,u} e^{i\Phi'^C_u} \ , \\
\sqrt{2} A(B^0_d \to \pi^0 K^0) &= P'_{ct} - \tilde{C}' \, e^{i\gamma} - P'_{EW} + \mathcal{A}'^{,comb} e^{i\Phi'} + \mathcal{A}'^{C,d} e^{i\Phi'^C_d} \ ,
\end{aligned}
\tag{22}
$$

while those for $B \to \pi\pi$ are

$$
\begin{aligned}
\sqrt{2} A(B^+ \to \pi^+ \pi^0) &= -\tilde{T} \, e^{i\gamma} - \tilde{C} \, e^{i\gamma} \ , \\
A(B^0_d \to \pi^+ \pi^-) &= -\tilde{T} \, e^{i\gamma} - P_{ct} \ , \\
\sqrt{2} A(B^0_d \to \pi^0 \pi^0) &= -\tilde{C} \, e^{i\gamma} + P_{ct} \ .
\end{aligned}
\tag{23}
$$

The amplitudes with tildes are related as in Eq. (19):

$$
\frac{\tilde{T}'}{\tilde{T}} = \frac{\tilde{C}'}{\tilde{C}} = \frac{\lambda}{1 - \lambda^2/2} \ .
\tag{24}
$$

There are now 14 theoretical unknown quantities, but 16 measurements. Thus, we can solve for the NP parameters with few discrete ambiguities. (In practice, one will fit for all parameters.)

Of course, we have assumed perfect SU(3) symmetry in this procedure [Eqs. (19) and (24)]. However, we know that there may be significant SU(3)-breaking effects. In Ref. [31], it was noted that factorization appears to hold for colour-allowed tree diagrams, so that $|\tilde{T}'/\tilde{T}| \simeq f_K/f_\pi$, but that the SU(3) breaking in the other relations would be left to experiment. In our case, NP is present in all $\bar{b} \to \bar{s}$ decays, and this will mask any SU(3)-breaking effects. One possibility is to assume that the ratios of magnitudes of amplitudes are known, but no assumption is made about the strong phases. That is, we write

$$
\left| \frac{\tilde{T}'}{\tilde{T}} \right| = f_T \ , \quad \left| \frac{\tilde{C}'}{\tilde{C}} \right| = f_C \ , \quad \left| \frac{P_{ct}}{P'_{ct}} \right| = f_P \ .
\tag{25}
$$

The quantities f_T, f_C and f_P are calculated using some theoretical model (e.g. QCD factorization [25]), but all strong phases are taken to be additional theoretical unknowns. The problem here is that this adds two theoretical quantities to the procedure (two strong phases in $B \to \pi\pi$ decays), so we once again have 16 measurements and 16 unknowns. As discussed above, this leads to a large number of discretely-ambiguous solutions. For this reason, it is probably best to assume

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that the strong phases of primed and unprimed amplitudes are equal, as with perfect SU(3) symmetry, and that the magnitude ratios are given by Eq. (25). In this case the above procedure will yield the NP parameters, but with sizeable theoretical errors.

Above, we have concentrated on $B \to PP$ decays, where P is a pseudoscalar. However, the analysis holds equally for $B \to VV$ decays (V is a vector meson). In this case, an angular analysis must be performed. Note that we have argued that exchange- and annihilation-type contributions to the $B \to PP$ decays are expected to be negligible. However, in some approaches to hadronic B decays, such amplitudes may be chirally enhanced if there are pseudoscalars in the final state [9, 25]. On the other hand, such chiral enhancements are not present for VV final states, so this is a potential point in favour of $B \to VV$ decays. It has been recently claimed that annihilation terms can also be big in certain $B \to VV$ decays in spite of the lack of chiral enhancement [32]. As noted earlier, ultimately, the size of exchange and annihilation diagrams is an experimental question, and can be tested by the measurement of decays such as $B_d^0 \to D_s^+ D_s^-$ and $B_d^0 \to K^+ K^-$.

One can even apply the method to PV final states, but things are considerably more complicated in this case since the NP contributes differently to the PV and VP final states. In this case, one must correspondingly increase the number of NP parameters: there are now a total of 30 parameters in e.g. $B \to \pi K^*$ and $B \to \rho K$ decays. These parameters are to be fitted to 32 measurements (9 in $B \to \pi K^*$, 9 in $B \to \rho K$, 13 in $B \to \rho \pi$, β from $B_d^0(t) \to J/\psi K_S$). Thus, while one can solve for the NP parameters in principle, in practice the analysis is very complicated.

We can do better with the $B \to \pi K / B \to \pi \pi$ method if we perform a semi-model-independent analysis by making an assumption about the general form of $\mathcal{A}'$'s. We illustrate this below.

3.1 Isospin-conserving new physics

One possibility is to make the general assumption that the new physics is isospin-conserving. For example, this occurs in NP models in which the gluonic penguin operators have an enhanced chromomagnetic moment [12]. In this case,

$$\mathcal{A}'^{,u} e^{i\Phi'_u} = \mathcal{A}'^{,d} e^{i\Phi'_d} \ , \ \ \mathcal{A}'^{C,u} e^{i\Phi'^C_u} = \mathcal{A}'^{C,d} e^{i\Phi'^C_d} \equiv \mathcal{A}'^C e^{i\Phi'^C} \ . \tag{26}$$

This in turn implies that $\mathcal{A}'^{,comb} e^{i\Phi'}$ is zero, so that there is just one $\mathcal{A}'$ remaining. The $B \to \pi K$ amplitudes now take the form:

$$
\begin{aligned}
A(B^+ \to \pi^+ K^0) &= P'_{ct} + \mathcal{A}'^C e^{i\Phi'^C} \ , \\
\sqrt{2} A(B^+ \to \pi^0 K^+) &= -P'_{ct} - \tilde{T}' e^{i\gamma} - \tilde{C}' e^{i\gamma} - P'_{EW} - \mathcal{A}'^C e^{i\Phi'^C} \ , \\
A(B_d^0 \to \pi^- K^+) &= -P'_{ct} - \tilde{T}' e^{i\gamma} - \mathcal{A}'^C e^{i\Phi'^C} \ , \\
\sqrt{2} A(B_d^0 \to \pi^0 K^0) &= P'_{ct} - \tilde{C}' e^{i\gamma} - P'_{EW} + \mathcal{A}'^C e^{i\Phi'^C} \ .
\end{aligned}
\tag{27}
$$

There are now only 10 theoretical parameters: 4 amplitude magnitudes, 3 δ's, 2 ϕ's and one Φ. Recall that there are 9 measurements in $B \to \pi K$ alone. We therefore need only one additional measurement to be able to extract all parameters, including those related to NP: $\mathcal{A}'^C$ and Φ'^C. Ideally, in order to reduce discrete ambiguities, we would have two additional measurements. These can come from independent measurements of the SM phases (the 2 ϕ's). Thus, for this particular type of new physics, we do not need measurements in the $B \to \pi\pi$ system at all.

Indeed, it is not necssary to make any assumptions about the absence of new physics in decays with $\bar{b} \to \bar{d}$ penguins, though we must assume that the phase of B_d^0–$\overline{B_d^0}$ mixing is measured in $B_d^0(t) \to J/\psi K_s$. If we assume that NP is also present in $B \to \pi\pi$ decays, these amplitudes take the form

$$
\begin{aligned}
\sqrt{2} A(B^+ \to \pi^+\pi^0) &= -\tilde{T} e^{i\gamma} - \tilde{C} e^{i\gamma} + \mathcal{A}^{comb} e^{i\Phi} - \mathcal{A}^{C,u} e^{i\Phi_u^C} + \mathcal{A}^{C,d} e^{i\Phi_d^C} \,, \\
A(B_d^0 \to \pi^+\pi^-) &= -\tilde{T} e^{i\gamma} - P_{ct} - \mathcal{A}^{C,u} e^{i\Phi_u^C} \,, \\
\sqrt{2} A(B_d^0 \to \pi^0\pi^0) &= -\tilde{C} e^{i\gamma} + P_{ct} + \mathcal{A}^{comb} e^{i\Phi} + \mathcal{A}^{C,d} e^{i\Phi_d^C} \,,
\end{aligned}
\tag{28}
$$

where

$$
\mathcal{A}^{comb} e^{i\Phi} \equiv -\mathcal{A}^u e^{i\Phi_u} + \mathcal{A}^d e^{i\Phi_d} \,.
\tag{29}
$$

If the NP is isospin-conserving, this implies that

$$
\mathcal{A}^u e^{i\Phi_u} = \mathcal{A}^d e^{i\Phi_d} \,, \quad \mathcal{A}^{C,u} e^{i\Phi_u^C} = \mathcal{A}^{C,d} e^{i\Phi_d^C} \equiv \mathcal{A}^C e^{i\Phi^{NP,C}} \,.
\tag{30}
$$

In this case, $\mathcal{A}^{comb} e^{i\Phi}$ vanishes, and we are left with only one $\mathcal{A}$, namely $\mathcal{A}^C e^{i\Phi^{NP,C}}$.

Assuming perfect SU(3) symmetry, the $B \to \pi K$ and $B \to \pi\pi$ amplitudes are described by a total of 12 theoretical quantities. With 16 measurements, it is possible to extract all parameters, including those describing the NP in the $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ transitions.

4 $B \to V_1 V_2$ Decays

In this section we examine $B \to V_1 V_2$ decays in which $\overline{V}_1 \overline{V}_2 = V_1 V_2$. We consider decays which are described by the quark-level transitions $\bar{b} \to \bar{c}c\bar{s}$, $\bar{b} \to \bar{s}s\bar{s}$, or $\bar{b} \to \bar{s}d\bar{d}$. Within the SM such decays are dominated by a single weak decay amplitude, and their weak phase is essentially zero in the standard parametrization [1]. (As noted above, there are two significant contributions — the tree and penguin amplitudes — for decays described by $\bar{b} \to \bar{s}u\bar{u}$.) Since these are all $\bar{b} \to \bar{s}$ transitions, there are new-physics contributions. Note that, in this method, we make no assumptions about NP in $\bar{b} \to \bar{d}$ decays.

Suppose that the underlying new-physics model is such that the weak phase is universal to all NP operators. As discussed in the introduction, this holds for a

large number of NP models. In this case, the NP weak phase Φ_q will be helicity-independent. Taking into account the fact that the NP strong phase is negligible, the decay amplitude for each of the three possible helicity states may be written as

$$
\begin{aligned}
A_\lambda \equiv Amp(B \to V_1 V_2)_\lambda &= a_\lambda e^{i\delta_\lambda} + \mathcal{A}_\lambda^q e^{i\Phi_q} , \\
\bar{A}_\lambda \equiv Amp(\bar{B} \to V_1 V_2)_\lambda &= a_\lambda e^{i\delta_\lambda} + \mathcal{A}_\lambda^q e^{-i\Phi_q} ,
\end{aligned}
\tag{31}
$$

where a_λ and $\mathcal{A}_\lambda^q$ represent the helicity-dependent SM and NP amplitudes, respectively, the δ_λ are the SM strong phases, and the helicity index λ takes the values $\{0, \|, \perp\}$. Using CPT invariance, the full decay amplitudes can be written as

$$
\begin{aligned}
\mathcal{A} &= Amp(B \to V_1 V_2) = A_0 g_0 + A_\| g_\| + i\, A_\perp g_\perp , \\
\bar{\mathcal{A}} &= Amp(\bar{B} \to V_1 V_2) = \bar{A}_0 g_0 + \bar{A}_\| g_\| - i\, \bar{A}_\perp g_\perp ,
\end{aligned}
\tag{32}
$$

where the g_λ are the coefficients of the helicity amplitudes written in the linear polarization basis. The g_λ depend only on the angles describing the kinematics [33].

Using the above equations, we can write the time-dependent decay rates as

$$
\Gamma(\overset{(-)}{B}(t) \to V_1 V_2) = e^{-\Gamma t} \sum_{\lambda \le \sigma} \Big(\Lambda_{\lambda\sigma} \pm \Sigma_{\lambda\sigma} \cos(\Delta M t) \mp \rho_{\lambda\sigma} \sin(\Delta M t) \Big) g_\lambda g_\sigma .
\tag{33}
$$

Thus, by performing a time-dependent angular analysis of the decay $B(t) \to V_1 V_2$, one can measure 18 observables. These are:

$$
\Lambda_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 + |\bar{A}_\lambda|^2), \qquad \Sigma_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 - |\bar{A}_\lambda|^2),
$$

$$
\Lambda_{\perp i} = -\text{Im}(A_\perp A_i^* - \bar{A}_\perp \bar{A}_i^*), \qquad \Lambda_{\|0} = \text{Re}(A_\| A_0^* + \bar{A}_\| \bar{A}_0^*),
$$

$$
\Sigma_{\perp i} = -\text{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*), \qquad \Sigma_{\|0} = \text{Re}(A_\| A_0^* - \bar{A}_\| \bar{A}_0^*),
$$

$$
\rho_{\perp i} = \text{Re}\Big(e^{-i\phi_M^q}[A_\perp^* \bar{A}_i + A_i^* \bar{A}_\perp]\Big), \qquad \rho_{\perp\perp} = \text{Im}\Big(e^{-i\phi_M^q} A_\perp^* \bar{A}_\perp\Big),
$$

$$
\rho_{\|0} = -\text{Im}\Big(e^{-i\phi_M^q}[A_\|^* \bar{A}_0 + A_0^* \bar{A}_\|]\Big), \qquad \rho_{ii} = -\text{Im}\Big(e^{-i\phi_M^q} A_i^* \bar{A}_i\Big),
\tag{34}
$$

where $i = \{0, \|\}$. As in Sec. 2.1, ϕ_M^q is the weak phase factor associated with B_q^0–$\bar{B}_q^0$ mixing. Note that the signs of the various $\rho_{\lambda\lambda}$ terms depend on the CP-parity of the various helicity states. We have chosen the sign of ρ_{ii} to be -1, which corresponds to the final state ϕK^*.

For measuring new-physics parameters, the key point is the following. There are a total of six amplitudes describing $B \to V_1 V_2$ and $\bar{B} \to V_1 V_2$ decays [Eq. (31)]. At best one can measure the magnitudes and relative phases of these six amplitudes. Thus, of the 18 observables, only 11 are independent. However, these observables are a function of only 11 theoretical parameters[8]: three a_λ's, three $\mathcal{A}_\lambda^q$'s, ϕ_M^q, Φ_q,

---

[8]If the NP weak phase is assumed to helicity-dependent, then there are more theoretical parameters than there are measurements, and we cannot solve the system of equations.

and the three strong phases δ_λ. In addition, as discussed in Sec. 2.1, ϕ_M^q can be measured independently, so we effectively have 11 equations in 10 unknowns. The solution will have discrete ambiguities, but many of these can be removed using the additional observables.

Thus, as advertised, if new physics is found, it is possible to *measure* the NP parameters via a time-dependent angular analysis of $B \to V_1 V_2$ decays. To be specific, $\mathcal{A}_\lambda^s$ and Φ_s can be extracted from $B_d^0 \to \phi K^{*0}$ or $B_s^0 \to \phi\phi$. The decays $B_d^0 \to K^{*0}\rho^0$ and $B_s^0 \to K^{*0}\bar{K}^{*0}$ can be used to measure $\mathcal{A}_\lambda^d$ and Φ_d. (In the decay $B_d^0 \to K^{*0}\rho^0$, there is a small theoretical error due to the neglect of the colour-suppressed $\bar{b} \to \bar{s}u\bar{u}$ tree contribution.) Finally, measurements of the decay $B_s^0 \to D_s^{*+}D_s^{*-}$ can be used to obtain $\mathcal{A}_\lambda^c$ and Φ_c.

Note that this analysis is done within the context of a single $\bar{b} \to \bar{s}$ $B \to V_1 V_2$ decay. In this case, as in the $B \to \pi K / B \to \pi\pi$ method with isospin-conserving new physics, no assumption is necessary about the absence of new physics in decays with $\bar{b} \to \bar{d}$ penguins. The only assumption needed is that the phase of B_q^0–$\bar{B}_q^0$ mixing, which may be affected by NP, can be extracted from $B_d^0(t) \to J/\psi K_S$ or $B_s^0(t) \to J/\psi \eta$.

4.1 Explicit Solution

Under the assumption that ϕ_M^q is known independently, we can construct an analytic solution (this follows closely the analysis of Ref. [15]). In terms of the theoretical parameters, the explicit expressions for the observables are as follows:

$$
\begin{aligned}
\Lambda_{\lambda\lambda} &= a_\lambda^2 + (\mathcal{A}_\lambda^q)^2 + 2a_\lambda \mathcal{A}_\lambda^q \cos\delta_\lambda \cos\Phi_q \ , \\
\Sigma_{\lambda\lambda} &= 2a_\lambda \mathcal{A}_\lambda^q \sin\delta_\lambda \sin\Phi_q \ , \\
\Lambda_{\perp i} &= 2\left[a_\perp \mathcal{A}_i^q \cos\delta_\perp - a_i \mathcal{A}_\perp^q \cos\delta_i \right] \sin\Phi_q \ , \\
\Lambda_{\|0} &= 2\left[a_\| a_0 \cos(\delta_\| - \delta_0) + a_\| \mathcal{A}_0^q \cos\delta_\| \cos\Phi_q + a_0 \mathcal{A}_\|^q \cos\delta_0 \cos\Phi_q + \mathcal{A}_\|^q \mathcal{A}_0^q \right] , \\
\Sigma_{\perp i} &= -2\left[a_\perp a_i \sin(\delta_\perp - \delta_i) + a_\perp \mathcal{A}_i^q \sin\delta_\perp \cos\Phi_q - a_i \mathcal{A}_\perp^q \sin\delta_i \cos\Phi_q \right] , \\
\Sigma_{\|0} &= 2\left[a_\| \mathcal{A}_0^q \sin\delta_\| + a_0 \mathcal{A}_\|^q \sin\delta_0 \right] \sin\Phi_q \ , \\
\rho_{ii} &= a_i^2 \sin 2\phi_M^q + 2a_i \mathcal{A}_i^q \cos\delta_i \sin(2\phi_M^q + \Phi_q) + (\mathcal{A}_i^q)^2 \sin(2\phi_M^q + 2\Phi_q), \\
\rho_{\perp\perp} &= -a_\perp^2 \sin 2\phi_M^q - 2a_\perp \mathcal{A}_\perp^q \cos\delta_\perp \sin(2\phi_M^q + \Phi_q) - (\mathcal{A}_\perp^q)^2 \sin(2\phi_M^q + 2\Phi_q), \\
\rho_{\perp i} &= 2\left[a_i a_\perp \cos(\delta_i - \delta_\perp) \cos 2\phi_M^q + a_\perp \mathcal{A}_i^q \cos\delta_\perp \cos(2\phi_M^q + \Phi_q) \right. \\
&\quad \left. + a_i \mathcal{A}_\perp^q \cos\delta_i \cos(2\phi_M^q + \Phi_q) + \mathcal{A}_i^q \mathcal{A}_\perp^q \cos(2\phi_M^q + 2\Phi_q) \right] , \\
\rho_{\|0} &= 2\left[a_0 a_\| \cos(\delta_\| - \delta_0) \sin 2\phi_M^q + a_\| \mathcal{A}_0^q \cos\delta_\| \sin(2\phi_M^q + \Phi_q) \right. \\
&\quad \left. + a_0 \mathcal{A}_\|^q \cos\delta_0 \sin(2\phi_M^q + \Phi_q) + \mathcal{A}_0^q \mathcal{A}_\|^q \sin(2\phi_M^q + 2\Phi_q) \right] .
\end{aligned}
\tag{35}
$$

For $B \to V_1 V_2$ decays, the analogue of the usual direct CP asymmetry a_{dir}^{CP} is the helicity-dependent quantity $a_\lambda^{dir} \equiv \Sigma_{\lambda\lambda}/\Lambda_{\lambda\lambda}$. We define the related quantity

$$y_\lambda \equiv \sqrt{1 - \Sigma_{\lambda\lambda}^2/\Lambda_{\lambda\lambda}^2} \; . \tag{36}$$

Similarly, the value of $\sin 2\phi_M^q$ measured in $B \to V_1 V_2$ decays can depend on the helicity of the final state: $\rho_{\lambda\lambda}$ can be recast in terms of a measured weak phase $(2\phi_M^q)_\lambda^{meas}$, defined as

$$\sin 2 \left(\phi_M^q\right)_\lambda^{meas} \equiv \frac{\pm \rho_{\lambda\lambda}}{\sqrt{\Lambda_{\lambda\lambda}^2 - \Sigma_{\lambda\lambda}^2}} \; , \tag{37}$$

where the $+$ $(-)$ sign corresponds to $\lambda = 0, \|$ $(\perp)$.

Using the expressions for $\Lambda_{\lambda\lambda}$, $\Sigma_{\lambda\lambda}$ and $(\phi_M^q)_\lambda^{meas}$ above, one can express a_λ and $\mathcal{A}_\lambda^q$ as follows [15]:

$$\begin{aligned}
2\, a_\lambda^2 \sin^2 \Phi_q &= \Lambda_{\lambda\lambda} \left(1 - y_\lambda \cos(\theta_\lambda - 2\Phi_q)\right) \; , \\
2\left(\mathcal{A}_\lambda^q\right)^2 \sin^2 \Phi_q &= \Lambda_{\lambda\lambda} \left(1 - y_\lambda \cos\theta_\lambda\right) \; ,
\end{aligned} \tag{38}$$

where $\theta_\lambda \equiv (2\phi_M^q)_\lambda^{meas} - 2\phi_M^q$. Using these expressions, along with those for $\Lambda_{\lambda\lambda}$ and $\Sigma_{\lambda\lambda}$ [Eq. (35)], we can solve for $\tan\delta_\lambda$:

$$\tan\delta_\lambda = \frac{\Sigma_{\lambda\lambda} \sin 2\Phi_q}{\Lambda_{\lambda\lambda}\left(-2\cos^2\Phi_q + y_\lambda \cos(\theta_\lambda - 2\Phi_q) + y_\lambda \cos\theta_\lambda\right)} \; . \tag{39}$$

This equation expresses δ_λ in terms of observables and Φ_q.

How one proceeds further depends on which other observables are available. Suppose that $\Lambda_{\perp i}$ and $\Sigma_{\perp i}$ have been measured. These two observables can be expressed as [15]

$$\begin{aligned}
\Sigma_{\perp i} &= P_i P_\perp \left[(\xi_\perp \sigma_i - \xi_i \sigma_\perp) \cos\Delta_i - (1 + \xi_i \xi_\perp + \sigma_i \sigma_\perp) \sin\Delta_i\right] \; , \\
\Lambda_{\perp i} &= P_i P_\perp \left[(\xi_\perp - \xi_i) \cos\Delta_i - (\sigma_i + \sigma_\perp) \sin\Delta_i\right] \; ,
\end{aligned} \tag{40}$$

where $\Delta_i \equiv \delta_\perp - \delta_i$, and

$$\begin{aligned}
P_\lambda^2 &\equiv \Lambda_{\lambda\lambda}(1 - y_\lambda \cos(2\theta_\lambda - 2\Phi_q)) \; , \\
\xi_\lambda &\equiv \frac{\Lambda_{\lambda\lambda}\, y_\lambda \sin(2\theta_\lambda - 2\Phi_q)}{P_\lambda^2} \; , \\
\sigma_\lambda &\equiv \frac{\Sigma_{\lambda\lambda}}{P_\lambda^2} \; .
\end{aligned} \tag{41}$$

Eqs. (40) can be solved for Δ_i:

$$\tan\Delta_i = \frac{(\xi_\perp \sigma_i - \xi_i \sigma_\perp)\,\Lambda_{\perp i} - \Sigma_{\perp i}\,(\xi_\perp - \xi_i)}{(1 + \xi_i \xi_\perp + \sigma_i \sigma_\perp)\,\Lambda_{\perp i} - \Sigma_{\perp i}\,(\sigma_i + \sigma_\perp)} \; . \tag{42}$$

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This expresses $\tan \Delta_i$ in terms of observables and Φ_q. However, we can also write

$$\tan \Delta_i = \frac{\tan \delta_\perp - \tan \delta_i}{1 + \tan \delta_\perp \tan \delta_i} \ . \tag{43}$$

Eqs. (39), (42) and (43) can then be combined to give a single equation as a function of Φ_q. This can be solved to get the new-physics weak phase, which will permit the measurement of the remaining theoretical parameters.

4.2   Are the NP strong phases negligible?

The time-dependent angular analysis also allows us to *test* the assumption that the NP strong phases are negligible. Assume that the $B \to V_1 V_2$ amplitudes contain (helicity-dependent) NP strong phases Δ_λ^q. In this case Eq. (31) can be written

$$
\begin{aligned}
A_\lambda \equiv Amp(B \to V_1 V_2)_\lambda &= a_\lambda e^{i\delta_\lambda} + \mathcal{A}_\lambda^q e^{i\Phi_q} e^{i\Delta_\lambda^q} \ , \\
\bar{A}_\lambda \equiv Amp(\bar{B} \to V_1 V_2)_\lambda &= a_\lambda e^{i\delta_\lambda} + \mathcal{A}_\lambda^q e^{-i\Phi_q} e^{i\Delta_\lambda^q} \ .
\end{aligned}
\tag{44}
$$

There are now 13 theoretical parameters, and only 11 observables. However, since the expressions for the observables in terms of parameters are nonlinear, one can obtain *bounds* on the various theoretical quantities [15]. In particular, one can constrain the two NP strong phase differences $\Delta_\perp^q - \Delta_i^q$, $i = 0, \|$. If either of these bounds is inconsistent with zero, this will invalidate the assumption of negligible NP strong phases.

5   Discussion

In Ref. [8] it was argued that the NP strong phases are negligible relative to those of the SM. In this case, all NP matrix elements for a given $\bar{b} \to \bar{s}q\bar{q}$ process ($q = u, d, s, c$) can be summed into a single effective NP operator, with amplitude $\mathcal{A}^q$ and corresponding weak phase Φ_q. In the previous sections, we have examined three different methods for measuring these NP parameters. They are: (i) the comparison of $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ penguin B decays [8], (ii) the combined measurement of $B \to \pi K$ and $B \to \pi\pi$ decays, and (iii) the time-dependent angular analysis of $B \to V_1 V_2$ decays. All three methods have their particular advantages and disadvantages.

There are several B decay pairs to which the B-penguin method can be applied. Depending on which pair is used, the s-, d- and c-quark NP parameters can be obtained with a theoretical error of about 5–15%. The u-quark NP parameters $\mathcal{A}^u$ and Φ_u can also be measured, but with a much larger theoretical error. A key assumption is that all SM weak phases have already been measured. Also, most pairs of modes involve time-dependent B_s^0 decays, which are hard to measure.

The $B \to \pi K / B \to \pi\pi$ method probes only the d- and u-quark NP parameters, and with a large ($\gtrsim 25\%$) theoretical error. Still, the theoretical error on $\mathcal{A}^u$ and

Φ_u might well be smaller than in the method with B penguin decays. The main advantage of this method is that the analysis can be done now. Ref. [31] analyzed these B decays within the SM; it is straightforward to include NP parameters in the analysis. It is also possible to adapt the method to include NP effects in $B \to \pi\pi$ if one makes some assumptions about the nature of the NP.

The $B \to V_1 V_2$ method has no theoretical error – flavour SU(3) is not required, and we make no assumption about new physics in $\bar{b} \to \bar{d}$ transitions. However, this method only allows us to measure the s-, d- and c-quark NP parameters, and it applies only to those new-physics models in which the NP weak phase is universal. Furthermore, it is very difficult experimentally. On the other hand, it can be used to *test* the assumption of negligible NP strong phases.

At several points previously, we have argued that it is important to measure all the NP parameters, and in as many different ways a possible. In the following, we show how different NP models lead to different patterns of NP parameters. Thus, the measurement of the NP parameters can rule out certain models and point towards others.

There has already been a great deal of theoretical work discussing various NP models which can explain the apparent discrepancy in the Belle measurement of $\sin 2\beta$ in $B_d^0(t) \to \phi K_s$ [34, 35, 36]. Our aim here is not to produce an exhaustive analysis of such NP models. Instead we consider only two, and show that the measurement of the NP parameters can distinguish between them.

5.1 *Z*-mediated FCNC's

The first new-physics model we consider is Z-mediated (or Z'-mediated) FCNC's [10]. This model has received much attention as a potential explanation of the $B_d^0(t) \to \phi K_s$ result [34]. The $Zb\bar{s}$ FCNC coupling which leads to the $\bar{b} \to \bar{s}$ transitions is parametrized by the independent parameter U_{sb}^z:

$$\mathcal{L}_{FCNC}^z = -\frac{g}{2 \cos \theta_W} U_{sb}^z \, \bar{s}_L \gamma^\mu b_L Z_\mu \ . \tag{45}$$

Note that the FCNC involves only left-handed s and b quarks. These couplings are effectively new contributions to the electroweak penguin operators of the SM.

The new-physics weak phase arises because U_{sb}^z can be complex. However, because this parameter is universal, the weak phase of all NP operators will be the same. This model therefore predicts the equality of all NP weak phases Φ_q ($q = u, d, s, c$). If this condition is not found to be satisfied, the model is ruled out.

5.2 Supersymmetry with R-parity breaking

In supersymmetric (SUSY) models with R-parity breaking, the relevant part of the R-parity breaking piece is given by

$$W_{\not{R}} = \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2}\lambda''_{i[jk]} U^c_i D^c_j D^c_k \ . \tag{46}$$

Here L_i and Q_i are the left-handed lepton and quark doublet superfields, respectively, and U_i and D_i are the left-handed quark singlet chiral superfields, where i, j, k are generation indices and c denotes a charge conjugate field.

In the above, the λ' and λ'' couplings violate lepton number and baryon number, respectively. The non-observation of proton decay imposes very stringent conditions on the simultaneous presence of both couplings [37]. One therefore assumes the existence of either L-violating couplings or B-violating couplings, but not both.

Now, $\lambda''_{i[jk]}$ is antisymmetric in the last two indices. Thus, there are no B-violating couplings which can lead to the $\bar{b} \to \bar{s}s\bar{s}$ decay necessary to explain the $B^0_d(t) \to \phi K_S$ result of Belle. We will therefore concentrate only on L-violating couplings. (For a discussion of R-parity violation and $B^0_d(t) \to \phi K_S$, see Ref. [35].) In terms of four-component Dirac spinors, the L-violating couplings are given by [38]

$$\begin{aligned}
\mathcal{L}_{\lambda'} &= -\lambda'_{ijk} \left[\tilde{\nu}^i_L \bar{d}^k_R d^j_L + \tilde{d}^j_L \bar{d}^k_R \nu^i_L + (\tilde{d}^k_R)^* (\bar{\nu}^i_L)^c d^j_L \right. \\
&\qquad\qquad \left. -\tilde{e}^i_L \bar{d}^k_R u^j_L - \tilde{u}^j_L \bar{d}^k_R e^i_L - (\tilde{d}^k_R)^* (\bar{e}^i_L)^c u^j_L \right] + h.c.
\end{aligned} \tag{47}$$

From this Lagrangian, we see that there are R-parity-violating contributions to all $\bar{b} \to \bar{s}$ transitions [39]. There is a single contribution to each of the decays $\bar{b} \to \bar{s}u\bar{u}$ and $\bar{b} \to \bar{s}c\bar{c}$:

$$\begin{aligned}
L^u_{eff} &= -\frac{\lambda'_{i12}\lambda'^*_{i13}}{2m^2_{\tilde{e}_i}} \bar{u}_\alpha \gamma_\mu \gamma_L u_\beta \, \bar{s}_\beta \gamma_\mu \gamma_R b_\alpha \ , \\
L^c_{eff} &= -\frac{\lambda'_{i22}\lambda'^*_{i23}}{2m^2_{\tilde{e}_i}} \bar{c}_\alpha \gamma_\mu \gamma_L c_\beta \, \bar{s}_\beta \gamma_\mu \gamma_R b_\alpha \ .
\end{aligned} \tag{48}$$

For $\bar{b} \to \bar{s}d\bar{d}$, there are four terms:

$$\begin{aligned}
L^d_{eff} &= \frac{\lambda'_{i11}\lambda'^*_{i23}}{m^2_{\tilde{\nu}_i}} \bar{d}\gamma_L d \, \bar{s}\gamma_R b + \frac{\lambda'_{i32}\lambda'^*_{i11}}{m^2_{\tilde{\nu}_i}} \bar{d}\gamma_R d \, \bar{s}\gamma_L b \\
&\quad - \frac{\lambda'_{i12}\lambda'^*_{i13}}{2m^2_{\tilde{\nu}_i}} \bar{d}_\alpha \gamma_\mu \gamma_L d_\beta \, \bar{s}_\beta \gamma_\mu \gamma_R b_\alpha - \frac{\lambda'_{i31}\lambda'^*_{i21}}{2m^2_{\tilde{\nu}_i}} \bar{d}_\alpha \gamma_\mu \gamma_R d_\beta \, \bar{s}_\beta \gamma_\mu \gamma_L b_\alpha \ .
\end{aligned} \tag{49}$$

Finally, the relevant Lagrangian for the $\bar{b} \to \bar{s}s\bar{s}$ transition is

$$L^s_{eff} = \frac{\lambda'_{i32}\lambda'^*_{i22}}{m^2_{\tilde{\nu}_i}} \bar{s}\gamma_R s \, \bar{s}\gamma_L b + \frac{\lambda'_{i22}\lambda'^*_{i23}}{m^2_{\tilde{\nu}_i}} \bar{s}\gamma_L s \, \bar{s}\gamma_R b \ . \tag{50}$$

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From the above expressions we can deduce the following predictions of R-parity-violating SUSY models. First, in general, all four NP parameters are present and are unrelated to one another. Second, since there is only a single term contributing to each of $\bar{b} \to \bar{s}u\bar{u}$ and $\bar{b} \to \bar{s}c\bar{c}$ transitions, the measured values of Φ_u and Φ_c should be independent of the decay pairs considered. On the other hand, since there is more than one contribution to both $\bar{b} \to \bar{s}d\bar{d}$ and $\bar{b} \to \bar{s}s\bar{s}$, the values of Φ_d and Φ_s will in general be process-dependent. Also, these will differ from Φ_u and Φ_c. Should this pattern of NP weak phases not be found experimentally, we can either rule out or constrain this model of new physics.

The above two examples illustrate that, indeed, different NP models lead to different patterns of the NP parameters $\mathcal{A}^q$ and Φ_q. Thus, the knowledge of the NP parameters will allow us to discriminate among various models, and rule certain ones out entirely. In order to (partially) identify the NP, it will be important to measure its parameters in as many different ways and decay modes as possible. A complete analysis of NP parameters would therefore include their measurement using all three of the methods described in the previous sections.

6 Conclusions

The main purpose of the study of CP violation in the B system is to look for physics beyond the SM. There are now many theoretical signals of such new physics, and in fact there are several experimental hints of NP in decays involving $\bar{b} \to \bar{s}$ penguin amplitudes. Still, the conventional thinking was that the *identification* of NP could only be done at future high-energy colliders, where the new particles could be directly produced. However, recently it was shown that it is possible to *measure* the NP parameters in B decays [8]. The key observation is that the strong phases associated with the NP operators are negligible compared to those of the SM. In this case, all NP matrix elements for a given $\bar{b} \to \bar{s}q\bar{q}$ process ($q = u, d, s, c$) can be summed into a single effective NP operator, with amplitude $\mathcal{A}^q$ and corresponding weak phase Φ_q. These NP parameters can be measured, allowing a partial identification of the new physics.

In this paper we have discussed three methods of measuring the NP parameters. In most cases, it is assumed that the new physics contributes only to decays with large $\bar{b} \to \bar{s}$ penguin amplitudes, while decays involving $\bar{b} \to \bar{d}$ penguins are not affected. The first method, initially proposed in Ref. [8], employs the comparison of time-dependent $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ penguin B decays. The second uses the combined measurement of $B \to \pi K$ and $B \to \pi\pi$ decays. The third requires the time-dependent angular analysis of $B \to V_1 V_2$ decays.

The three methods can be used to probe different NP parameters, and with different theoretical errors. The B-penguin method allows us to obtain the s-, d- and c-quark NP parameters with a theoretical error of about 5–10%. The u-quark

NP parameters can also be measured, but with a much larger theoretical error. The $B \to \pi K / B \to \pi\pi$ method probes only the d- and u-quark NP parameters, but with a large ($\gtrsim 25\%$) theoretical error. The $B \to V_1 V_2$ method has no theoretical error, but it applies only to NP models with a universal weak phase, and it only allows the measurement of the s-, d- and c-quark NP parameters.

The three methods also have different levels of experimental difficulty. Most pairs of modes in the B-penguin method involve time-dependent B_s^0 decays, which are hard to measure, and the time-dependent angular analysis of $B \to V_1 V_2$ decays is very difficult experimentally. On the other hand, the $B \to \pi K / B \to \pi\pi$ method can be performed with present data.

Ideally, a full analysis of NP parameters would use all three methods. Then all the NP parameters $\mathcal{A}^q$ and Φ_q can be measured in many different ways, and using various decay modes. In general, different NP models lead to different patterns of the NP operators. The knowledge of the NP parameters will thus allow us to discriminate among various models and partially identify the new physics, before the direct production of new particles at high-energy colliders.

Appendix

In this Appendix, we present a more detailed discussion of the claim that the NP strong phases are negligible. To understand our argument, it is crucial to examine where the SM and NP strong phases come from. Note that this argument is not dependent on any particular calculational framework. However, it will be useful to demonstrate it within a particular approach for calculating nonleptonic decays. We will follow the method of QCD factorization [25], also known as the BBNS approach.

To be more specific let us condider $B \to K\pi$ decays. The starting point for calculations of this decay is the SM effective hamiltonian for B decays [40]:

$$H_{eff}^q = \frac{G_F}{\sqrt{2}}[V_{fb}V_{fq}^*(c_1 O_{1f}^q + c_2 O_{2f}^q) - \sum_{i=3}^{10}(V_{tb}V_{tq}^* c_i^t)O_i^q] + h.c. \ , \qquad (A. 1)$$

where the superscript t indicates the internal top quark, f can be the u or c quark, and q can be either a d or s quark. The operators O_i^q are defined as

$$\begin{aligned}
O_{f1}^q &= \bar{q}_\alpha \gamma_\mu L f_\beta \bar{f}_\beta \gamma^\mu L b_\alpha \ , & O_{2f}^q &= \bar{q}\gamma_\mu L f \bar{f} \gamma^\mu L b \ , \\
O_{3,5}^q &= \bar{q}\gamma_\mu L b \bar{q}' \gamma^\mu L(R) q' \ , & O_{4,6}^q &= \bar{q}_\alpha \gamma_\mu L b_\beta \bar{q}'_\beta \gamma^\mu L(R) q'_\alpha \ , \qquad (A. 2) \\
O_{7,9}^q &= \frac{3}{2}\bar{q}\gamma_\mu L b e_{q'} \bar{q}' \gamma^\mu R(L) q' \ , & O_{8,10}^q &= \frac{3}{2}\bar{q}_\alpha \gamma_\mu L b_\beta e_{q'} \bar{q}'_\beta \gamma^\mu R(L) q'_\alpha \ ,
\end{aligned}$$

in which $R(L) = 1 \pm \gamma_5$, and q' is summed over u, d, s, c. O_2 and O_1 are the tree-level and QCD-corrected operators, respectively. O_{3-6} are the strong gluon-induced penguin operators, and operators O_{7-10} are due to γ and Z exchange (electroweak penguins), and "box" diagrams at loop level.

First, we consider the SM alone. In particular, we consider the QCD penguin operators $O_{4,6}$ which contribute dominantly to the SM penguin amplitude for $B \to K\pi$. (Note that $V_{tb}V_{ts}^*$ in H_{eff} can be expressed in terms of $V_{cb}^* V_{cs}$ and $V_{ub}^* V_{us}$ using CKM unitarity.) Consider the $O_{4,6}$ operators with weak phase $V_{cb}^* V_{cs}$. (The same arguments apply to contributions with weak phase $V_{ub}^* V_{us}$.) The SM penguin amplitude can obtain a strong phase from two types of rescattering, which we call class I and class II:

- In class I, there is rescattering from the tree-level operators with Wilson coefficients $C_1 \approx 1$. The operator has the same weak phase as $O_{4,6}$ and leads to a $\bar{b} \to \bar{c}c\bar{s}$ transition. This will contribute to a final state like $K\pi$ only through rescattering and will consequently generate a strong phase. The matrix element in question is $\langle K\pi|O_1|B\rangle$, and is called P_c. In the BBNS picture this rescattering is represented by the penguin function $G(x)$ and is proportional to $C_1 \approx 1$. The $G(x)$ generates the dominant strong phase.

  In the usual approach, including that of BBNS, the contribution of the tree operator is combined with the matrix element of the $O_{4,6}$ operator in the

quantity $a_{4,6}$ to represent the SM penguin contribution. The scattering from the tree operator is the dominant rescattering effect and is responsible for the strong phase of the SM penguin amplitude. Note that the strength of the $O_{4,6}$ operators, $C_{4,6}$, is only $\sim 4\%$. Thus, even though rescattering costs α_s, it is enhanced by the large Wilson coefficient C_1, which is 25 times as large as C_4. The rescattered tree amplitude is therefore roughly the same size as the matrix element of the $O_{4,6}$ operator, so that it can generate a significant strong phase in the BBNS approach [25].

- In class II, one can have rescattering from the operators $O_{4,6}$ themselves: $\langle K\pi|O_{4,6}|B\rangle$. This is represented by BBNS's $g(x)$ and $G(x)$, and is proportional to C_4 or C_6. However, as mentioned above, the size of C_4 and C_6 is only about 4%. Unlike Class I rescattering, there are no operators with large Wilson coefficients to produce an appreciable strong phase here. In other words, the strength of these operators is much smaller than $C_1 \approx 1$. So this class of rescattering is subdominant.

Now we consider the presence of NP. At m_b, one can separate the effective Hamiltonian into two pieces: O_4 (say), which is the SM operator with the SM Wilson coefficient, and $\widetilde{O}_4$ which contains all the NP contributions. We assume that $\widetilde{O}_4$ and O_4 are of similar size. The strong phase now comes from three sources: rescattering from O_1 (class I), O_4 (class II) and $\widetilde{O}_4$ (class II). The rescattering effects from O_1 and O_4 are simply those corresponding to the SM, whose strong phase can be large. However, the strong phase of the matrix element of $\widetilde{O}_4$ comes *only* from rescattering from the NP operator itself: $\langle K\pi|\widetilde{O}_4|B\rangle$. That is, the NP rescattering is only of the Class II type. Hence, the NP strong phases are *subdominant*. To a first approximation, we therefore ignore the NP strong phases compared to the SM strong phases.

To demonstrate this point numerically let us work with a specific model of new physics, R-parity violating SUSY, that was considered in Sec. 5. In this model the Lagrangian for $\bar{b} \to \bar{s}u\bar{u}$ and $\bar{b} \to \bar{s}c\bar{c}$ transitions is :

$$
\begin{aligned}
L_{eff}^u &= -\frac{\lambda'_{i12}\lambda'^*_{i13}}{2m_{e_i}^2}\bar{u}_\alpha\gamma_\mu\gamma_L u_\beta\,\bar{s}_\beta\gamma_\mu\gamma_R b_\alpha\ , \\
L_{eff}^c &= -\frac{\lambda'_{i22}\lambda'^*_{i23}}{2m_{e_i}^2}\bar{c}_\alpha\gamma_\mu\gamma_L c_\beta\,\bar{s}_\beta\gamma_\mu\gamma_R b_\alpha\ .
\end{aligned}
\tag{A. 3}
$$

For the decays $B \to K\pi$, the term in L_{eff}^u can contribute directly or through rescattering of the u-quark loop in the BBNS method. The quantity r, which is the ratio of the imaginary part of the amplitude relative to its real part, is given by [25]

$$
r = \frac{C_F\alpha_s(m_b)}{4\pi N_c}Im[\hat{G}_K(s_u)]
$$

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$$\hat{G}_K(s) = \int_0^1 dx G(s_u, 1-x)\Phi_p^K(x)$$

$$G(s,x) = -4\int_0^1 u(1-u)ln[s-u(1-u)x] , \tag{A. 4}$$

where $C_F = (N_c^2 - 1)/(2N_c)$, $s_u = (m_u/m_b)^2$ and $\Phi_p^K(x)$ is the light-cone distribution(LCD) of the kaon . Using the asymptotic LCD and the fact that $s_u = 0$ to a very good approximation, we obtain

$$r = \frac{C_F\alpha_s(m_b)}{4\pi N_c}\frac{2\pi}{3} \approx 0.015 \tag{A. 5}$$

where we have taken $\alpha_s(m_b) = 0.2$. Clearly the NP strong phase from Eq. (A. 5) is $\sim 1^0$, and is negligible. The factor of $\alpha_s(m_b)/(4\pi) \approx 0.016$ comes from the strong coupling constant and the loop factor, and produces a significant suppression of the rescattering. Only in the SM can this rescattering be significant, as it is proportional to the large Wilson coefficient C_1. There are also rescattering effects such as the vertex contributions in the BBNS approach, which will generate new operators with different colour structure than L_{eff}^u. However we expect these operators to be suppressed by the factor of $\alpha_s(m_b)/(4\pi) \approx 0.016$ relative to L_{eff}^u and hence to be negligible. The Lagrangian L_{eff}^c can contribute to $K\pi$ final state only through rescattering and will be suppressed by the same factor $\alpha_s(m_b)/(4\pi)$ relative to L_{eff}^u. It can therefore also be neglected.

For $\bar{b} \to \bar{s}d\bar{d}$, there are four terms in the Lagrangian:

$$\begin{aligned}
L_{eff}^d &= \frac{\lambda'_{i11}\lambda'^*_{i23}}{m_{\tilde{\nu}_i}^2}\bar{d}\gamma_L d\,\bar{s}\gamma_R b + \frac{\lambda'_{i32}\lambda'^*_{i11}}{m_{\tilde{\nu}_i}^2}\bar{d}\gamma_R d\,\bar{s}\gamma_L b \\
&\quad - \frac{\lambda'_{i12}\lambda'^*_{i13}}{2m_{\tilde{\nu}_i}^2}\bar{d}_\alpha\gamma_\mu\gamma_L d_\beta\,\bar{s}_\beta\gamma_\mu\gamma_R b_\alpha - \frac{\lambda'_{i31}\lambda'^*_{i21}}{2m_{\tilde{\nu}_i}^2}\bar{d}_\alpha\gamma_\mu\gamma_R d_\beta\,\bar{s}_\beta\gamma_\mu\gamma_L b_\alpha . \tag{A. 6}
\end{aligned}$$

For the decays $B \to K\pi$, the terms in L_{eff}^d can contribute directly or through rescattering of the d-quark loop in the BBNS method. The terms involving vector operators will have the same rescatterieng as L_{eff}^u and so the resulting strong phases can once again be neglected. The scalar operators can only have electroweak rescattering though the the exchange of a photon or a Z and will therefore be even smaller.

Finally, the relevant Lagrangian for the $\bar{b} \to \bar{s}s\bar{s}$ transition is

$$L_{eff}^s = \frac{\lambda'_{i32}\lambda'^*_{i22}}{m_{\tilde{\nu}_i}^2}\bar{s}\gamma_R s\,\bar{s}\gamma_L b + \frac{\lambda'_{i22}\lambda'^*_{i23}}{m_{\tilde{\nu}_i}^2}\bar{s}\gamma_L s\,\bar{s}\gamma_R b . \tag{A. 7}$$

The terms in L_{eff}^s can contribute to the $K\pi$ final state only through rescattering. They will therefore be suppressed by the factor $\alpha_s(m_b)/(4\pi)$ relative to $L_{eff}^{u,d}$, and

hence can be neglected. Note that the operators $\bar{s}b\bar{b}b$ and $\bar{s}b\bar{t}t$ can also contribute through rescattering but will have no imaginary contribution as the gluon momentum is below the threshold of $b\bar{b}$ and $t\bar{t}$ production.

We have therefore shown in general terms, and within a specific model of NP, that the NP strong phases are small compared to those of the SM, using the BBNS approach. As mentioned above, this result is independent of the specific model of NP, as well as of the method used to calculate nonleptonic decays.

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